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New solitary wave solutions to tunnel diode model against its numerical solutions

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New solitary wave solutions to tunnel diode model against its numerical solutions

M R Nady[1]* , Emad H M Zahran[1] and Reda A Ibrahim[1]

[1] Department of Basic Science, Faculty of Engineering at Shoubra, Benha University, Cairo, Egypt.

*E-mail: mohamed.ragab@feng.bu.edu.eg

Abstract. In this work, we will concentrate on constructing novel forms of soliton solution for the Lonngren Wave Equation. The Lonngren Wave Equation is important in areas where understanding wave phenomena is critical, including engineering, physics, and applied mathematics. It allows for analysis and prediction of wave behaviour under various physical conditions. These forms of soliton solution will be obtained using two of the recent efficient analytic techniques, one of them is the Riccati-Bernoulli Sub-OD Equation method, which is not obeys to the principle of homogenous balance. The other analytic method which obeys the homogenous balance principle is the extended simple equation method. Besides the two analytic methods, we introduce the approximate solutions corresponding to the soliton solutions obtained before by the mentioned analytic methods using the numerical technique called the Haar Wavelet Method. With the help of Mathematica program, the 2D and 3D graphs are considered to explain the physical and geometric interpretations of the obtained results. The obtained solitons are of the kind periodic parabolic soliton solution, bright soliton solution, dark soliton solution, kink soliton solution. Our results are obtained for the first time, and they are important and effective compared to the results obtained by other authors for the same problem.

1.Introduction

The development of global communication systems is urgent and important. The communication technologies challenge for transmit and receive real-time information which connect geographical divides with each others and enhance collaboration across all industries. That is clear in critical infrastructure such as healthcare, education, and emergency response systems, .In order to ensure all the world is always connected, the signals must be stable and strong. Nonlinear partial differential equations (NLPDEs) help to describe the complex natural phenomena into mathematical model. NLPDEs are considered as the bridge between practical and theoretical studies in various fields such as fluid mechanics, materials science, solid-state physics, mathematical biology, optical fibers, circuit analysis, control theory, plasma physics besides electrical and communication engineering [1]. The mathematical solutions for those NLPDEs give the traveling waves into the system. This helps us to check and develop our results from simulation before implementation of the system in real life. In contrast to linear Partial Differential Equations (PDEs), NLPDEs exhibit complex behaviors such as shock waves and solitary waves, which introduce significant challenges in their analysis. One of the most intriguing solutions to NLPDEs is the soliton solution, which can be defined as a stable and localized waveform that preserves its shape while propagating at a constant velocity [2]. These solitons play a critical role in various engineering systems and continue to be a focus of extensive research in the study of nonlinear dynamics [3].Their analysis helps us to study and choose the desired behavior for system while designing stage. In recent years, these equations have been solved using numerous methods to provide analytical solutions [4]. For example, Inverse Scattering Transform Method [5], GKM [6]. Multiple traveling wave soliton solutions have been established using different methods such as Extended Direct Algebraic Method [7], the expansion method [8], Paul-Painlevé Approach[9], Sine Cosine Method [10], Modified Khater Method [11], the Generalized Exponential Rational Function Method [12] and the (Ψ − Φ)-Expansion Method [13].Also, dynamical system approaches have been used like Modified Simple Equation Technique and the Sine-Gordon Expansion Techniques[14].These techniques provide a geometric framework for analyzing wave propagation in nonlinear systems, deepening our comprehension of the fundamental dynamics involved. LWE can be utilized to study the propagation of sound waves in various media, which is essential for designing acoustic systems in concert halls. In seismology, they allow for the analysis of seismic waves generated by earthquakes, contributing to a better understanding of the Earth's internal structure and improving the prediction of seismic events. In electromagnetics, the results obtained describe the behavior of electromagnetic waves, including light, radio waves, and microwaves, supporting the development of antennas, optical fibers, and communication systems. In addition, Engineers can also apply these findings to analyze and design structures subjected to dynamic loads, such as vibrations and oscillations, which is essential in civil, mechanical, and aerospace engineering. In medical imaging, technologies like ultrasound and magnetic resonance imaging depend on wave propagation principles, and understanding the wave equation is fundamental to advancing and interpreting these imaging techniques. In optics, these wave solutions play a vital role in explaining phenomena like diffraction, interference, and the polarization of light. This insight is crucial for developing and refining optical devices, lenses, and imaging technologies. This expertise is applied in diverse industries, such as construction, electronics, and aerospace. In geophysics, our findings can help geophysicists explore phenomena like seismic surveys for oil and gas, groundwater detection, and environmental monitoring [15]. In this research, we focus on an application of communication engineering. LWE describes the behavior of transmission electrical signals in a type of semiconductor material called tunnel diode or the mechanisms underlying energy storage in circuits featuring electric charge [16–18].

Consider an infinite ideal transmission line composed of typical sections contain capacitor (C_s) and inductor (L) as shown in figure 1. The unit of inductor is henry per unit length while capacitor is farad per unit length. The following equations derived by using Kirchhoff's Laws.

$$
\begin{aligned}
&\frac{\partial I}{\partial x}+\frac{\partial[VD_N(V)]}{\partial t}=0,\\
&\frac{\partial V}{\partial x}+L\frac{\partial I_1}{\partial t}=0,\\
&\frac{\partial^2 V}{\partial x\partial t}+\frac{1}{C_S}I_2=0,\\
&I_2=I-I_1.
\end{aligned}
\tag{1.1}
$$

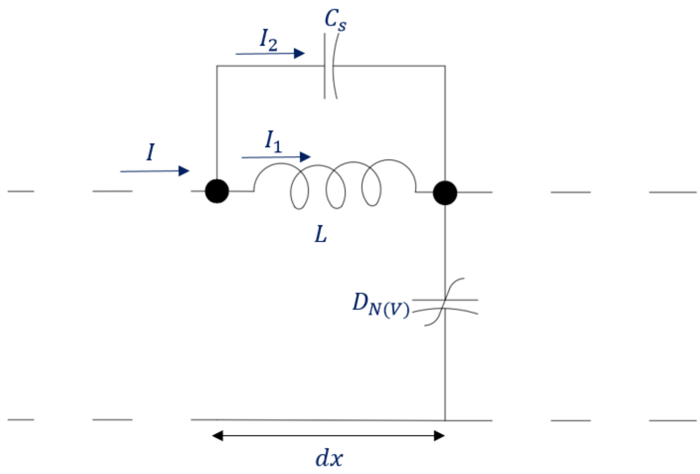


Figure 1. The configuration of the tunnel diode at a typical segment of nonlinear transmission line

The nonlinear diode is described by $D_N(V)$. We can use Eq. (1.1) to deduce the following equation which describes the wave equation of voltage as:

$$\frac{\partial^4 V}{\partial x^2 \partial t^2} + \omega_o^2 \frac{\partial^2 V}{\partial x^2} - \frac{1}{C_S}\frac{\partial^2 [V D_N(V)]}{\partial t^2} = 0. \quad (1.2)$$

Where $\omega_o^2 = \dfrac{1}{LC_S}$.

Transforming Eq. (1.2) to dimensionless formula of non-linear 4th-order NLPDE, [19,20]:

$$\frac{\partial^4 u}{\partial x^2 \partial t^2} - \alpha \frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial x^2} + 2\beta[(\frac{\partial u}{\partial t})^2 + u\frac{\partial^2 u}{\partial t^2}] = 0. \quad (1.3)$$

Eq. (1.3) can be reduced to the following form:

$$(u_{xx} - \alpha u + \beta u^2)_{tt} + u_{xx} = 0. \quad (1.4)$$

Which is called LWE. Where, α and β are defined as arbitrary constants. Additionally, the constant β acts as a coefficient that defines the nonlinearity term. The temporal and spatial variables are denoted by t and x respectively and the wave function u depends on these variables.

In our research, we are seeking to solve LWE by using two analytic solitary wave methods, which have never been used before in this context such as the ESEM [21–23], the RBSODEM [24–26], and numerical method called HWM [27–29]. The next sections will clarify these methods and the gain results.

2.ESEM

To obtain the solution of any NLPDE using ESEM, let us assume the general form of them as

$$F(u, u_x, u_t, u_{xx}, u_{tt}, \ldots) = 0. \quad (2.1)$$

Where F in terms of the successive partial derivatives of $u(x,t)$, when Eq. (2.1) surrenders to the transformation $u(x,t) = u(\xi)$, $\xi = \omega(x - ct)$, it becomes ordinary differential equations (ODE) in the form

$$Z(u, u', u'', u''', \ldots) = 0. \tag{2.2}$$

Where Z is in terms of the total derivatives of $u(\xi)$.From the point of view of the ESEM, the general form of solution is

$$u(\xi) = \sum_{i=-M}^{M} A_i \varphi(\xi)^i. \tag{2.3}$$

The function $\varphi(\xi)$ will be determined from

$$\varphi'(\xi) = a_0 + a_1\varphi + a_2\varphi^2 + a_3\varphi^3. \tag{2.4}$$

The positive integers .M . in Eq. (2.3) can be determined by the homogeneous balance principle, while A_i are constants that will be determined later, and arbitrary constants a_0, a_1 and a_2 will generate these cases for Eq. (2.4).

First family ($a_1 = a_3 = 0$): it will be transformed to the Riccati equation, which has the following solutions

$$\varphi(\xi) = \frac{\sqrt{a_0 a_2}}{a_2} \tan\left(\sqrt{a_0 a_2}\,(\xi + \xi_0)\right), a_0 a_2 > 0. \tag{2.5}$$

$$\varphi(\xi) = \frac{\sqrt{-a_0 a_2}}{a_2} \tanh\left(\sqrt{-a_0 a_2}\,\xi - \frac{\rho_1 \ln \xi_0}{2}\right), a_0 a_2 < 0, \xi > 0, \rho_1 = \pm 1. \tag{2.6}$$

Second family ($a_0 = a_3 = 0$): it will be transformed into the Bernoulli equation, which has the following solutions

$$\varphi(\xi) = \frac{a_1 e^{a_1(\xi+\xi_0)}}{1 - a_2 e^{a_1(\xi+\xi_0)}}, a_1 > 0. \tag{2.7}$$

$$\varphi(\xi) = \frac{-a_1 e^{a_1(\xi+\xi_0)}}{1 + a_2 e^{a_1(\xi+\xi_0)}}, a_1 < 0. \tag{2.8}$$

Moreover, the general solution can be described as

$$\varphi(\xi) = -\frac{1}{a_2}\left(a_1 - \sqrt{4a_1a_2 - a_1^2}\tan\left(\frac{\sqrt{4a_1a_2 - a_1^2}}{2}(\xi + \xi_0)\right)\right), 4a_1a_2 > a_1^2, a_2 > 0. \tag{2.9}$$

$$\varphi(\xi) = \frac{1}{a_2}\left(a_1 + \sqrt{4a_1a_2 - a_1^2}\tanh\left(\frac{\sqrt{4a_1a_2 - a_1^2}}{2}(\xi + \xi_0)\right)\right), 4a_1a_2 > a_1^2, a_2 < 0. \tag{2.10}$$

Where ξ_0 is the constant of integration.

By inserting Eq. (2.3) into Eq. (2.2) and collect the equivalence for various powers of φ^i.This emerges an algebraic system. By solving it, these parameters will be extracted, then final solution is easily obtained.

2.1 ESEM for solving LWE:

Using transformation $u(x,t)=u(\xi)$, $\xi(x,t)=\omega(x-ct)$, the LWE Eq.(1.4) will be converted to the following nonlinear ODE

$$\omega^2\left(2c^2\beta\phi'(\xi)^2+\left(1-c^2\alpha+2c^2\beta\phi(\xi)\right)\phi''(\xi)+c^2\omega^2\phi^{(4)}(\xi)\right)=0. \tag{2.11}$$

Integrating the above equation twice with respect to ξ We get

$$\phi(\xi)\left(1-c^2\alpha+c^2\beta\phi(\xi)\right)+c^2\omega^2\phi''(\xi)=0. \tag{2.12}$$

According to Eq. (2.3) and applying balance principle ($M=2$) the solution of Eq. (2.12) can be written as

$$\phi(\xi)=A_0+\frac{A_{-2}}{\varphi(\xi)^2}+\frac{A_{-1}}{\varphi(\xi)}+A_1\varphi(\xi)+A_2\varphi(\xi)^2. \tag{2.13}$$

2.1.1 For the first family ($a_1=a_3=0$), Eq.(2.4) becomes

$$\varphi'(\xi)=a_0+a_2\varphi(\xi)^2 \tag{2.14}$$

Using Eq. (2.13) and its second differentiation with aid of Eq. (2.14) into Eq. (2.12) and equating the similar coefficients of $\varphi(\xi)^i$ to zero, we obtain the following system of equations

$$\left.\begin{array}{l}
2c^2\omega^2a_2^2A_{-2}+A_0-c^2\alpha A_0+c^2\beta A_0^2+2c^2\beta A_{-1}A_1+2c^2\omega^2a_0^2A_2+2c^2\beta A_{-2}A_2=0\\
A_{-1}-c^2\alpha A_{-1}+2c^2\omega^2a_0a_2A_{-1}+2c^2\beta A_{-1}A_0+2c^2\beta A_{-2}A_1=0\\
A_{-2}-c^2\alpha A_{-2}+8c^2\omega^2a_0a_2A_{-2}+c^2\beta A_{-1}^2+2c^2\beta A_{-2}A_0=0\\
2c^2\omega^2a_0^2A_{-1}+2c^2\beta A_{-2}A_{-1}=0\\
6c^2\omega^2a_0^2A_{-2}+c^2\beta A_{-2}^2=0\\
A_1-c^2\alpha A_1+2c^2\omega^2a_0a_2A_1+2c^2\beta A_0A_1+2c^2\beta A_{-1}A_2=0\\
c^2\beta A_1^2+A_2-c^2\alpha A_2+8c^2\omega^2a_0a_2A_2+2c^2\beta A_0A_2=0\\
2c^2\omega^2a_2^2A_1+2c^2\beta A_1A_2=0\\
6c^2\omega^2a_2^2A_2+c^2\beta A_2^2=0
\end{array}\right\}. \tag{2.15}$$

The previous system has many solutions; due to the similarity and correspondence of the obtained results we will choose the following results

$$A_{-1}=0,A_{-2}=0,a_0=\frac{1-c^2\alpha}{4c^2\omega^2a_2},A_0=\frac{3\left(-1+c^2\alpha\right)}{2c^2\beta},A_1=0,A_2=-\frac{6\omega^2a_2^2}{\beta}. \tag{2.16}$$

$$A_{-1}=0,A_{-2}=-\frac{6\omega^2a_0^2}{\beta},a_2=\frac{1-c^2\alpha}{16c^2\omega^2a_0},A_0=\frac{3\left(-1+c^2\alpha\right)}{4c^2\beta},A_1=0,A_2=-\frac{3\left(-1+c^2\alpha\right)^2}{128c^4\beta\omega^2a_0^2}. \tag{2.17}$$

$$A_{-1}=0,A_{-2}=-\frac{6\omega^2a_0^2}{\beta},a_2=\frac{1-c^2\alpha}{4c^2\omega^2a_0},A_0=\frac{3\left(-1+c^2\alpha\right)}{2c^2\beta},A_1=0,A_2=0. \tag{2.18}$$

For the first result

When substituting in Eq. (2.16) with values $\beta=1,\omega=1,a_2=3,\alpha=0.5,c=0.3$. It becomes

$$A_{-1}=0,A_{-2}=0,A_1=0,a_0=0.8843,A_0=-15.91667,A_2=-54. \quad (2.19)$$

According to previously obtained values, using Eq.(2.5) , the $\varphi(\xi)$ will be

$$\varphi(\xi)=0.5429\tan(1.6287(\xi+1)). \quad (2.20)$$

So, the solution to Eq.(2.12) is:

$$\phi(\xi)=-15.91667-15.9174\left(\tan\left[1.6287(\xi+1)\right]\right)^2.$$

$$\phi(x,t)=-15.91667-15.9174\left(\tan\left[1.6287(1-0.3t+x)\right]\right)^2. \quad (2.21)$$

The second result

When substituting in Eq. (2.17) with values $\beta=1,\omega=1,a_0=3,\alpha=0.5,c=0.3$. it will be

$$A_{-1}=0,A_1=0,A_{-2}=-54,a_2=0.2211,A_0=-7.9583,A_2=-0.29322. \quad (2.22)$$

According to the above values the function $\varphi(\xi)$ from Eq. (2.5) is

$$\varphi(\xi)=3.6838\tan(0.8144(\xi+1)). \quad (2.23)$$

Using the previous equation to obtain the final solution

$$\phi(\xi)=-7.9583-3.9792\left(\cot\left(0.8144(\xi+1)\right)\right)^2-3.9792\left(\tan\left(0.8144(\xi+1)\right)\right)^2.$$

$$\phi(x,t)=-7.9583-3.9792\left(\cot\left(0.8144(1-0.3t+x)\right)\right)^2-3.9792\left(\tan\left(0.8144(1-0.3t+x)\right)\right)^2. \quad (2.24)$$

The third result

By the same method, When substituting in Eq. (2.18) with values $\beta=1,\omega=1,a_0=3,\alpha=0.5,c=0.3$. We get

$$A_{-1}=0,A_1=0,A_2=0,A_{-2}=-54,a_2=0.8843,A_0=-15.9167. \quad (2.25)$$

These values will lead to the same form of Eq. (2.5) from which, the final solution is:

$$\phi(\xi)=-15.9167-15.9174\left(\cot\left(1.6287(\xi+1)\right)\right)^2.$$

$$\phi(x,t)=-15.9167-15.9174\left(\cot\left(1.6287(1-0.3t+x)\right)\right)^2. \quad (2.26)$$

2.1.2 For the second family ($a_0=a_3=0$)

Substituting on Eq. (2.4). It becomes

$$\varphi'(\xi)\ =a_1\varphi+a_2\varphi^2. \quad (2.27)$$

By the same method of previous case using Eq. (2.13) and its second differentiation with aid with Eq. (2.27) into Eq. (2.12) then equating the similar various powers of φ^i to obtain the following system of equations

$$\left.\begin{array}{l}2c^2\omega^2a_2^2A_{-2}+c^2\omega^2a_1a_2A_{-1}+A_0-c^2\alpha A_0+c^2\beta A_0^2+2c^2\beta A_{-1}A_1+2c^2\beta A_{-2}A_2=0\\6c^2\omega^2a_1a_2A_{-2}+A_{-1}-c^2\alpha A_{-1}+c^2\omega^2a_1^2A_{-1}+2c^2\beta A_{-1}A_0+2c^2\beta A_{-2}A_1=0\\3c^2\omega^2a_1a_2A_1+c^2\beta A_1^2+A_2-c^2\alpha A_2+4c^2\omega^2a_1^2A_2+2c^2\beta A_0A_2=0\\A_{-2}-c^2\alpha A_{-2}+4c^2\omega^2a_1^2A_{-2}+c^2\beta A_{-1}^2+2c^2\beta A_{-2}A_0=0\\A_1-c^2\alpha A_1+c^2\omega^2a_1^2A_1+2c^2\beta A_0A_1+2c^2\beta A_{-1}A_2=0\\2c^2\omega^2a_2^2A_1+10c^2\omega^2a_1a_2A_2+2c^2\beta A_1A_2=0\\c^2\beta A_{-2}^2=0\\2c^2\beta A_{-2}A_{-1}=0\\6c^2\omega^2a_2^2A_2+c^2\beta A_2^2=0\end{array}\right\}. \quad (2.28)$$

The solution of this system are

$$A_{-1}=0, A_{-2}=0, a_1\to\frac{\sqrt{-1+c^2\alpha}}{c\omega}, A_0=0, A_1=-\frac{6\sqrt{-1+c^2\alpha}\,\omega a_2}{c\beta}, A_2=-\frac{6\omega^2a_2^2}{\beta}. \quad (2.29)$$

$$A_{-1}=0, A_{-2}=0, a_1=-\frac{\sqrt{-1+c^2\alpha}}{c\omega}, A_0=0, A_1=\frac{6\sqrt{-1+c^2\alpha}\,\omega a_2}{c\beta}, A_2=-\frac{6\omega^2a_2^2}{\beta}. \quad (2.30)$$

The first result

Substituting at Eq. (2.29) with values $\beta=1,\omega=1,a_2=10,\alpha=2,c=1$. It becomes

$$A_{-1}=0, A_{-2}=0, A_0=0, a_1=1, A_1=-60, A_2=-600. \quad (2.31)$$

Using the above the function $\varphi(\xi)$ takes the form Eq. (2.7) as

$$\varphi(\xi)=\frac{e^{\xi+1}}{1-10e^{\xi+1}}. \quad (2.32)$$

Then, the solution of Eq. (2.4) is

$$\phi(\xi)=-\frac{600e^{2(\xi+1)}}{\left(1-10e^{\xi+1}\right)^2}-\frac{60e^{\xi+1}}{1-10e^{\xi+1}}\to\phi(x,t)=-\frac{600e^{2-2t+2x}}{\left(1-10e^{1-t+x}\right)^2}-\frac{60e^{1-t+x}}{1-10e^{1-t+x}}. \quad (2.33)$$

The second solution

By the same method, when substituting at Eq. (2.30) with values $\beta=1,\omega=1,a_2=3,\alpha=0.5,c=3$ .It becomes

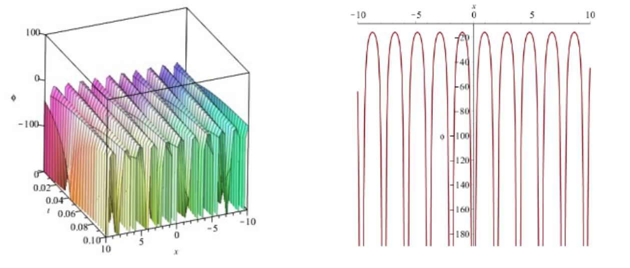
$$A_{-1}=0, A_{-2}=0, A_0=0, a_1=-0.6236, A_1=11.225, A_2=-54. \quad (2.34)$$

The solution of Eq. (2.27) satisfies Eq. (2.8) after using the above values.

$$\varphi(\xi)=\frac{0.6236e^{-0.6236(\xi+1)}}{1+3e^{-0.6236(\xi+1)}}. \quad (2.35)$$

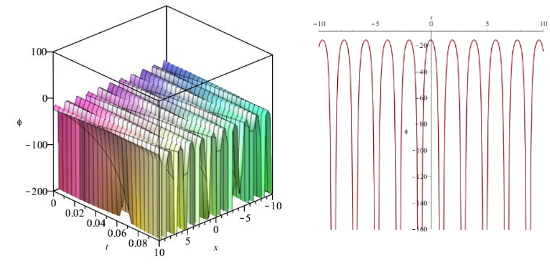
So, the solution of Eq. (2.4) is

$$\phi(\xi)=-\frac{21.e^{-1.2472(\xi+1)}}{\left(1+3e^{-0.6236(\xi+1)}\right)^2}+\frac{7.e^{-0.6236(\xi+1)}}{1+3e^{-0.6236(\xi+1)}}\to\phi(x,t)=-\frac{21.e^{-1.2472(1-3t+x)}}{\left(1+3e^{-0.6236(1-3t+x)}\right)^2}+\frac{7.e^{-0.6236(1-3t+x)}}{1+3e^{-0.6236(1-3t+x)}}. \quad (2.36)$$



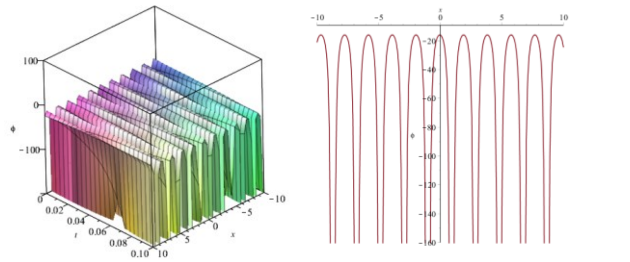
(a) (b)

Figure 2. (a) the 3D and (b) the 2D for first result of first family with $\beta=1, \omega=1, a_2=3, \alpha=0.5, c=0.3$
$A_{-1}=0, A_{-2}=0, A_1=0, a_0=0.8843, A_0=-15.91667, A_2=-54,$
$x=-10:10\, and\ t=0:0.1s$



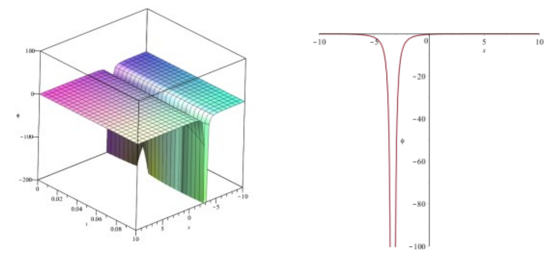
(a) (b)

Figure 3. (a) the 3D and (b) the 2D for second result of first family with
$\beta=1, \omega=1, a_0=3, \alpha=0.5, c=0.3$
$A_{-1}=0, A_1=0, A_{-2}=-54, a_2=0.2211, A_0=-7.9583,$
$A_2=-0.29322, x=-10:10\, and\ t=0:0.1s$



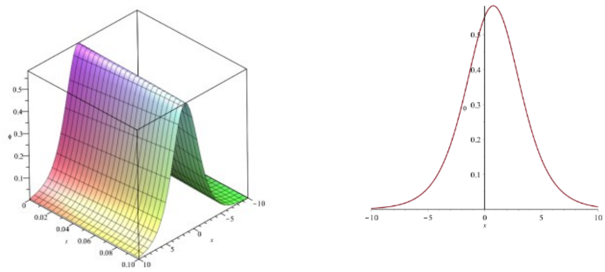
(a) (b)

Figure 4. (a) the 3D and (b) the 2D for third result of first family Eq. (2.26);
$x=-10:10\, and\ t=0:0.1s.$



(a) (b)

Figure 5. (a) the 3D and (b) the 2D for first result of second family Eq. (2.33) ;
$x=-10:10\, and\ t=0:0.1s.$



(a) (b)

Figure 6. (a) the 3D and (b) the 2D for first result of second family Eq. (2.36);
$x=-10:10\, and\ t=0:0.1s.$

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3.RBSUBODE

The RBSODM suggests the solution of any nonlinear differential equation in the form

$$v' = Av^{2-n} + Bv + Cv^{n}. \tag{3.1}$$

Where A, B, C, and n are constants to be determined later.

It can easily obtain the second derivative of Eq. (3.1) to be

$$v'' = AB(3-n)v^{2-n} + A^{2}(2-n)v^{3-2n} + nC^{2}v^{2n-1} + BC(n+1)v^{n} + (2AC+B^{2})v. \tag{3.2}$$

It is important to note that: When $AC\neq 0$ and $n = 0$, Eq.(3.1) tends to Riccati equation but When $A\neq 0$,
$C = 0$, and $n\neq 1$,it tends to a Bernoulli equation. i.e. the Riccati equation and Bernoulli equation are special cases of Eq.(3.1).

The solution of Eq.(3.1) depends on the values of A, B, C *and* $n.$ that has the following cases of solution.

Case 1: When $n = 1,$ the solution is:

$$v(\xi) = ke^{(A+B+C)\xi}. \tag{3.3}$$

Case 2: When $n \neq 1,$ $B = 0$ *and* $C = 0,$ the solution is:

$$v(\xi) = \left(A(n-1)(\xi+k)\right)^{\frac{1}{n-1}}. \tag{3.4}$$

Case 3: When $n \neq 1, B \neq 0$ *and* $C = 0,$ the solution is:

$$v(\xi) = \left(-\frac{A}{B} + ke^{B(n-1)\xi}\right)^{\frac{1}{n-1}}. \tag{3.5}$$

Case 4: When $n \neq 1, A \neq 0$ *and* $B^{2} - 4AC < 0,$ the solutions are:

$$v(\xi) = \left(-\frac{B}{2A} + \frac{\sqrt{4AC-B^{2}}}{2A}\tan\left(\frac{(1-n)\sqrt{4AC-B^{2}}}{2}(\xi+k)\right)\right)^{\frac{1}{1-n}}. \tag{3.6}$$

Or

$$v(\xi) = \left(-\frac{B}{2A} - \frac{\sqrt{4AC-B^{2}}}{2A}\cot\left(\frac{(1-n)\sqrt{4AC-B^{2}}}{2}(\xi+k)\right)\right)^{\frac{1}{1-n}}. \tag{3.7}$$

Case 5: When $n \neq 1, A \neq 0$ *and* $B^{2} - 4AC > 0,$ the solutions are:

$$v(\xi) = \left(-\frac{B}{2A} - \frac{\sqrt{B^{2}-4AC}}{2A}\coth\left(\frac{(1-n)\sqrt{B^{2}-4AC}}{2}(\xi+k)\right)\right)^{\frac{1}{1-n}}. \tag{3.8}$$

Or

$$v(\xi)=\left(-B-\frac{\sqrt{B^2-4AC}}{2A}\tanh\left(\frac{(1-n)\sqrt{B^2-4AC}}{2}(\xi+k)\right)\right)^{\frac{1}{1-n}}. \tag{3.9}$$

Case 6: When $n\neq 1, A\neq 0\, and\, B^2-4AC=0$, the solution is:

$$v(\xi)=\left(\frac{1}{A(n-1)(\xi+k)}-\frac{B}{2A}\right)^{\frac{1}{n-1}}. \tag{3.10}$$

Where k is an arbitrary constant.

3.1 RBSODEM for solving LWE

Substitute at Eq. (2.12) using Eq. (3.1) and Eq. (3.2)

$$\begin{gathered} c^2\omega^2\left(B-A(-2+n)\phi^{1-n}(\xi)+Cn\phi^{-1+n}(\xi)\right)\left(B\phi(\xi)+A\phi^{2-n}(\xi)+C\phi^{n}(\xi)\right)+ \\ \phi(\xi)\left(1-\alpha c^2+\beta c^2\phi(\xi)\right)=0. \end{gathered} \tag{3.11}$$

Taking n with suitable value.

$$BCc^2\omega^2+\left(1-\alpha c^2+B^2c^2\omega^2+2ACc^2\omega^2\right)\phi+\left(\beta c^2+3ABc^2\omega^2\right)\phi^2+2A^2c^2\omega^2\phi^3=0. \tag{3.12}$$

Equating the similar powers of $\phi(\xi)$ to the right-hand side's one can get the following system of equations

$$\begin{gathered} 1-\alpha c^2+B^2c^2\omega^2+2ACc^2\omega^2=0. \\ \beta c^2+3ABc^2\omega^2=0. \\ BCc^2\omega^2=0. \end{gathered} \tag{3.13}$$

By solving the above system, we get the two solutions

$$A=\pm\frac{\beta c\sqrt{-1+\alpha c^2}}{3\omega\left(-1+\alpha c^2\right)}, B=\mp\sqrt{\text{[illegible]}\,\frac{c^2}{\text{[illegible]}}}\, and\, C=0. \tag{3.14}$$

The first result can be obtain when substituting by $\alpha=2,\delta=1,\beta=2\, and\, \omega=1$ at the first solution of Eq. (3.14). Get the following

$$A=\frac{2}{3}, B=-1\, and\, C=0. \tag{3.15}$$

The result satisfy case 3.when taking k=1, Eq. (3.5) will be

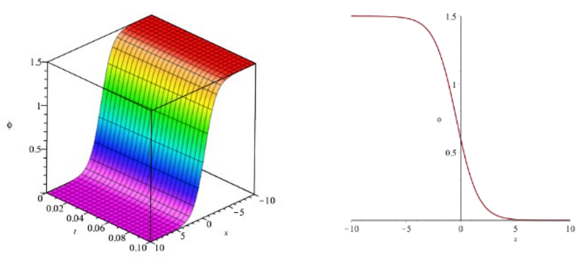
$$\phi(\xi)=\frac{1}{\frac{2}{3}+e^{\xi}}. \tag{3.16}$$

When taking the same values at Eq. (3.14) with the alternative signs to obtain the second result. Get the following

$$A=-\frac{2}{3}, B=1\, and\, C=0 \tag{3.17}$$

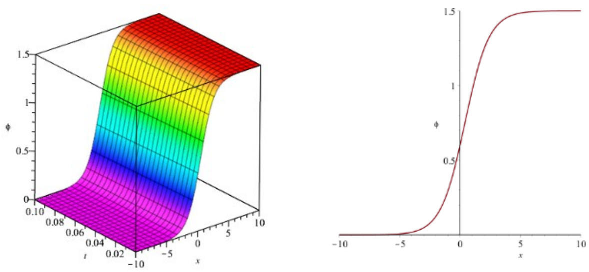
Which tends to the same case 3.when taking k=1.the second result is

$$\phi(\xi)=\frac{1}{\frac{2}{3}+e^{-\xi}}. \tag{3.18}$$



(a) (b)

Figure 7. (a) the 3D and (b) the 2D for first result Eq. (3.16) at $\alpha=2, c=1, \beta=2, \omega=1, y=0,$ $A=\frac{2}{3}, B=-1\, and\, C=0, x=-10:10\, and\ t=0:0.1s$



(a) (b)

Figure 8. (a) the 3D and (b) the 2D for second result Eq. (3.18) at $\alpha=2, c=1, \beta=2, \omega=1, y=0,$ $A=-\frac{2}{3}, B=1\, and\, C=0, x=-10:10\, and\ t=0:0.1s$

4.HWM

Wavelets are mathematical functions that oscillate from zero toward to peak and then return to zero. They are essential in a variety of applications due to their efficacy and simplicity.. It plays a vital role in many fields like signal processing and image compression .There are many kinds of wavelets as each of them come in various forms to specific purposes. Among these, the Haar Wavelet family holds a special place as the simplest and most basic type of wavelet. The named is related to Alfred Haar who introduce the concept in 1910. Haar wavelets are known for their ortho-normality, a property that ensures efficient decomposition and reconstruction of signals. The Haar wavelet, defined for $h_i(\xi) \in [0, 1]$, is as follows:

$$h_i(\xi)=\begin{cases} 1 & \text{for} \quad r \le \xi < s \\ -1 & \text{for} \quad s \le \xi < \gamma \\ 0 & otherwise \end{cases}. \tag{4.1}$$

The first integration of Haar Wavelet Function (4.1) is given by

$$p_i(\xi)=\begin{cases} \xi - r & \text{for} \quad r \le \xi < s \\ \gamma - \xi & \text{for} \quad s \le \xi < \gamma \\ 0 & otherwise \end{cases}. \tag{4.2}$$

The second integration is given by

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$$q_i(\xi)=\begin{cases} 0 & \text{for } 0\le\xi<r \\ \dfrac{(\xi-r)^2}{2} & \text{for } r\le\xi<s \\ \dfrac{1}{4m^2}-\dfrac{(\gamma-\xi)^2}{2} & \text{for } s\le\xi<\gamma \\ \dfrac{1}{4m^2} & \text{for } \gamma\le\xi<1 \end{cases}. \tag{4.3}$$

Generally, the remaining integrations can be obtained from

$$P_{n,i}(\xi)=\begin{cases} 0 & \text{for } 0\le\xi<p \\ \dfrac{(\xi-r)^n}{n!} & \text{for } r\le\xi<s \\ \dfrac{(\xi-r)^n-2(\xi-s)^n}{n!} & \text{for } s\le\xi<\gamma \\ \dfrac{(\xi-r)^n-2(\xi-s)^n+(\xi-\gamma)}{n!} & \text{for } \gamma\le\xi<1 \end{cases}. \tag{4.4}$$

Where, $r=\frac{k}{m}$, $s=\frac{k+0.5}{m}$, $\gamma=\frac{k+1}{m}$ Such that m, k ∈ ℤ, k is a translation parameters k=0, 1,…,m-1
$m=2^j$ is the wavelet level, j=0, 1,…,J and J is the maximal level of resolution
The index in h_i is gotten from i=m+k+1 giving the range from i=2 when m=1 to $i=2^{J+1}$ when $m=2^J$

$$h_1(\xi)=\begin{cases} 1 & \text{for } 0\le\xi<1 \\ 0 & otherwise \end{cases}. \tag{4.5}$$

Is called the scaling function or father wavelet, while

$$h_2(\xi)=\begin{cases} 1 & \text{for } 0\le\xi<0.5 \\ -1 & \text{for } 0.5\le\xi<1 \\ 0 & otherwise \end{cases}. \tag{4.6}$$

Called the mother function.

The main approach of Haar Wavelet Technique is convert the differential equations to system of algebraic equations within its coefficients are determined by solving them. Once getting the coefficients the solution of the differential equation can be easily obtained.

4.1 HWM procedures:

The HWM procedure suggests firstly, writing down the unknown function with max derivative in the form

$$u^{(k)}(\xi)=\sum_{i=1}^{M}a_i h_i(\xi). \tag{4.7}$$

Then**,** integrating the above equation k time's w.r.t ξ to obtain

$$u(\xi)=A_M^T H_M. \tag{4.8}$$

The arrival solution algebraic equation can be described as in matrix form where $A_M^T=[a_1,a_2,a_3,a_4,a_5,\ldots,a_M]$ is the Haar coefficients aiming to be determined later H_M is the matrix of Haar functions and M is the dimension of matrix given by M=2^{J+1}. After this, we satisfy and discretize eq. (4.8) at "Collocations points"

$$\Omega_\perp=\frac{\perp-0.5}{M}\quad \perp=1,2,3,\ldots,M. \tag{4.9}$$

which yields an algebraic system in the unknows Haar coefficients.

4.2 The HWM solution corresponding to ESEM

In this section, we will apply the HWM to solve Eq. (2.12).We seek to obtain the equivalent numerical solution to analytic second solution in the second family Eq. (2.36) that will be used to obtain the initial conditions.

According to this method, the solution can be expressed as

$$\phi(\xi)=\sum_{i=1}^{M}a_i q_i(\xi)+0.080198*\xi+0.551628. \tag{4.10}$$

The Haar coefficients a_i can be found by substituting Eq. (4.10) through Eq. (2.12)

$$\begin{gathered}25\sum_{i=1}^{M}a_i h_i(\xi)+16.081412\sum_{i=1}^{M}a_i q_i(\xi)+25\left(\sum_{i=1}^{M}a_i q_i(\xi)\right)^2\\+4.009917\xi\sum_{i=1}^{M}a_i q_i(\xi)+0.160794\xi^2+1.2897026\xi+1.263618=0.\end{gathered} \tag{4.11}$$

Satisfying the above equation at the collocation points, it becomes

$$\begin{gathered}25\sum_{i=1}^{M}a_i h_i(\xi_\Delta)+16.081412\sum_{i=1}^{M}a_i q_i(\xi_\Delta)+25\left(\sum_{i=1}^{M}a_i q_i(\xi_\Delta)\right)^2\\+4.009917\xi_\Delta\sum_{i=1}^{M}a_i q_i(\xi_\Delta)+0.160794{\xi_\Delta}^2+1.2897026\xi_\Delta+1.263618=0.\end{gathered} \tag{4.12}$$

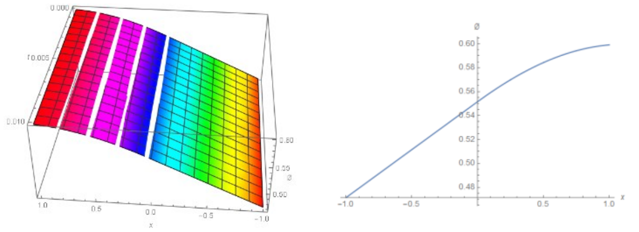
This will lead to an algebraic system of equations.

For $M=4\left(\xi_1=\frac{1}{8},\xi_2=\frac{3}{8},\xi_3=\frac{5}{8},\xi_4=\frac{7}{8}\right)$.

Satisfying Eq.(4.12) at four collocation points, we get a system of algebraic equations after solving it, the solution is

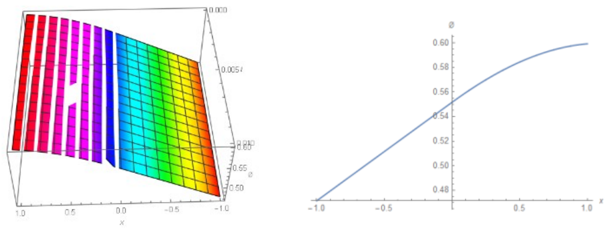
$$\phi(\xi)=0.5516+0.0802\xi-0.07082q_1(\xi)+0.0084q_2(\xi)+0.00557q_3(\xi)+0.00269q_4(\xi). \tag{4.13}$$

We work at various cases of M such as eight, sixteen and thirty-two. All the solutions are shown below figures.



(a) (b)

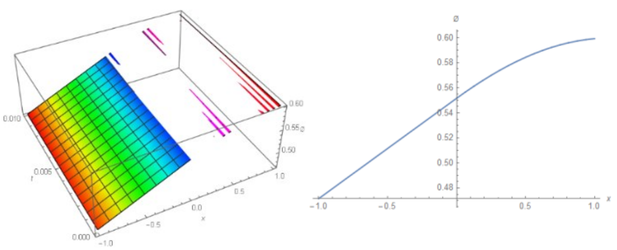
Figure 9. (a) the 3D and (b) the 2D for max level of resolution M=4 at $\alpha = 0.5, c = 5, \beta = 1, \omega = 1, a_1 = -0.07082,$ $a_2 = 0.00845,$ $a_3 = 0.0056, a_4 = 0.0027, x = -10:10 \; and \;\; t = 0:0.01s.$



(a) (b)

Figure 10. (a) the 3D and (b) the 2D for max level of resolution M=8 at $\alpha = 0.5, c = 5, \beta = 1, \omega = 1,$

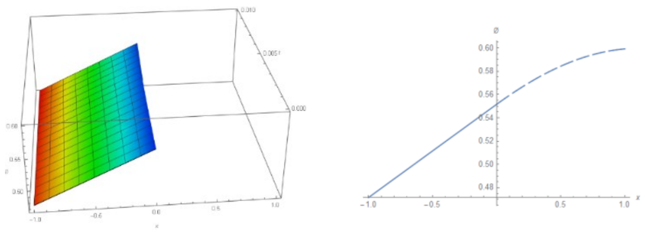
$a_1 = -0.07059453136413035,$ $a_2 = 0.0083, a_3 = 0.0056, a_4 = 0.00247,$ $a_5 = 0.00304, a_6 = 0.002498, a_7 = 0.0017,$ $a_8 = 0.00073, x = -10:10 \; and \;\; t = 0:0.01s.$



(a) (b)

Figure 11. (a) the 3D and (b) the 2D for max level of resolution M=16 at $\alpha = 0.5, c = 5, \beta = 1, \omega = 1,$

$a_1 = -0.0706, a_2 = 0.0083, a_3 = 0.00556, a_4 = 0.00247,$ $a_5 = 0.003, a_6 = 0.0025, a_7 = 0.001722, a_8 = 0.0007;$ $a_9 = 0.00157, a_{10} = 0.00146, a_{11} = 0.00133, a_{12} = 0.00116;$ $a_{13} =, 0.00097, a_{14} = 0.000748, a_{15} = 0.00049,$ $a_{16} = 0.00022, x = -10:10 \; and \;\; t = 0:0.01s.$



(a) (b)

Figure 12. (a) the 3D and (b) the 2D for max level of resolution M=32 at $\alpha = 0.5, c = 5, \beta = 1, \omega = 1,$

$a_1 = -0.07058, a_2 = 0.00827, a_3 = 0.0056, a_4 = 0.0025,$ $a_5 = 0.00304, a_6 = 0.0025, a_7 = 0.0017, a_8 = 0.000725,$ $a_9 = 0.00157, a_{10} = 0.00146, a_{11} = 0.0013, a_{12} = 0.001165,$ $a_{13} = 0.00097, a_{14} = 0.00075, a_{15} = 0.0005, a_{16} = 0.00022,$ $a_{17} = 0.000796, a_{18} = 0.00077, a_{19} = 0.00075, a_{20} = 0.00072,$ $a_{21} = 0.00068, a_{22} = 0.00064, a_{23} = 0.0006, a_{24} = 0.00056,$ $a_{25} = 0.000511, a_{26} = 0.00046, a_{27} = 0.0004, a_{28} = 0.000344,$ $a_{29} = 0.00028, a_{30} = 0.00022, a_{31} = 0.000147, a_{32} = 0.000076,$ $x = -10:10 \; and \;\; t = 0:0.01s.$

We can find a comparison between the analytic method and the numerical solutions at various maximum level of resolutions at the table below.

Table 1. Solutions comparison between ESEM and HWM at t=0.01.

x	At $t = 0.01s$.				
	ESEM	M=4	M=8	M=16	M=32
-0.6	0.483454	0.499499	0.499499	0.499499	0.499499
-0.4	0.507394	0.515539	0.515539	0.515539	0.515539
-0.2	0.52893	0.531579	0.531579	0.531579	0.531579
0	0.547507	0.547618	0.547618	0.547618	0.547618
0.2	0.562617	0.563019	0.563052	0.563058	0.56306
0.4	0.573825	0.576163	0.576225	0.576242	0.576247
0.6	0.580799	0.586634	0.586729	0.586753	0.586759
0.8	0.583326	0.59414	0.594269	0.5943	0.594308
1	0.581327	0.598475	0.598641	0.598675	0.598684

Table 2. Solutions comparison between ESEM and HWM at t=0.1.

x	At $t = 0.1s$.				
	ESEM	M=4	M=8	M=16	M=32
-0.6	0.423763	0.46341	0.46341	0.46341	0.46341
-0.4	0.451024	0.47945	0.47945	0.47945	0.47945
-0.2	0.477161	0.495489	0.495489	0.495489	0.495489
0	0.501612	0.511529	0.511529	0.511529	0.511529
0.2	0.523802	0.527569	0.527569	0.527569	0.527569
0.4	0.543169	0.543608	0.543608	0.543608	0.543608
0.6	0.55919	0.559364	0.559379	0.559385	0.559386
0.8	0.571409	0.573118	0.57317	0.573184	0.573188
1	0.579466	0.584279	0.584371	0.584393	0.584399

4.3 The HWM solution corresponding to RBSODEM

In this section, we will apply HWM to obtain the solution corresponding to the first solution of RBSODEM Eq. (3.16).the initial conditions can be obtained when substituting by $t = 0, x = 0$ at the first and second derivatives of the same equation.

$$\phi(\xi) = \sum_{i=1}^{M} a_i q_i(\xi) - \frac{9}{25}\xi + \frac{3}{5}. \tag{4.14}$$

Substituting by Eq. (4.14) into Eq. (2.12) . We get

$$7\sum_{i=1}^{M} a_i h_i(\xi) + \frac{7}{5}\sum_{i=1}^{M} a_i q_i(\xi) + 2\left(\sum_{i=1}^{M} a_i q_i(\xi)\right)^2 - \frac{36}{25}\xi\sum_{i=1}^{M} a_i q_i(\xi) + \frac{162}{625}\xi^2 - \frac{63}{125}\xi + \frac{3}{25} = 0. \tag{4.15}$$

When substituting with the collocation points. The above equation becomes

$$7\sum_{i=1}^{M} a_i h_i(\xi_\Delta) + \frac{7}{5}\sum_{i=1}^{M} a_i q_i(\xi_\Delta) + 2\left(\sum_{i=1}^{M} a_i q_i(\xi_\Delta)\right)^2 - \frac{36}{25}\xi_\Delta\sum_{i=1}^{M} a_i q_i(\xi_\Delta) + \frac{162}{625}{\xi_\Delta}^2 - \frac{63}{125}\xi_\Delta + \frac{3}{25} = 0. \tag{4.16}$$

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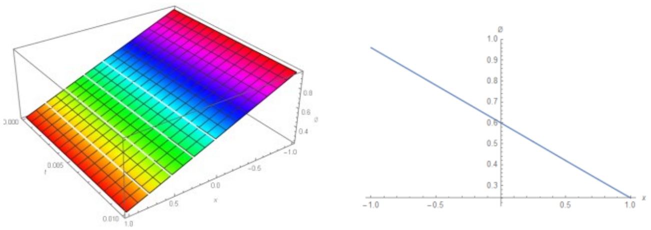
for $M=4\left(\xi_1=\frac{1}{8},\xi_2=\frac{3}{8},\xi_3=\frac{5}{8},\xi_4=\frac{7}{8}\right)$.

Satisfying Eq. (4.16) at four collocation points, an obtained system of algebraic equations is solving. We get

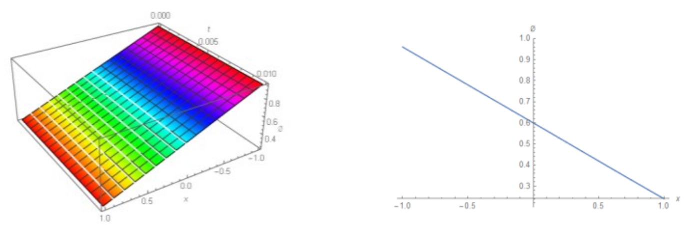
$$a_1=0.0067, a_2=-0.0087, a_3=-0.0067\ and\ a_4=-0.002. \quad (4.17)$$

By the same way, we can satisfy Eq. (4.16) at different values for M such as eight, sixteen and thirty-two collocation points then solving the obtained system of equations to get the Haar coefficients which required to get the solution Eq. (4.14).



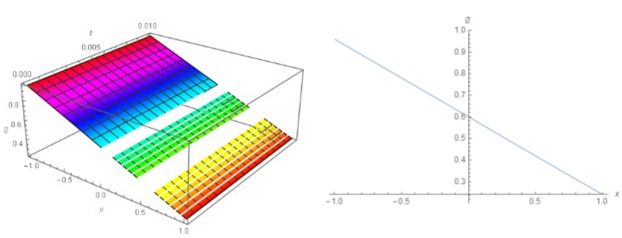
(a) (b)

Figure 13. (a) the 3D and (b) the 2D for Eq. (4.16) with M=4 at $\alpha=2, c=1, \beta=2, \omega=1, a_1=0.0067, a_2=-0.0087,$ $a_3=-0.0067, a_4=-0.002, x=-10:10\ and\ t=0:0.01s.$



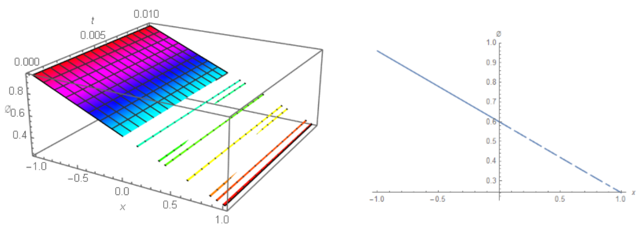
(a) (b)

Figure 14. (a) the 3D and (b) the 2D for Eq. (4.16) with M=8 at $\alpha=2, c=1, \beta=2, \omega=1,$ $a_1=0.0066, a_2=-0.0087, a_3=-0.0067, a_4=-0.002,$ $a_5=-0.0039, a_6=-0.0028, a_7=-0.00159, a_8=-0.00044,$ $x=-10:10\ and\ t=0:0.01s.$



(a) (b)

Figure 15. (a) the 3D and (b) the 2D for Eq. (4.16) with M=16 at $\alpha=2, c=1, \beta=2, \omega=1,$ $a_1=0.0066; a_2=-0.0087; a_3=-0.00671; a_4=-0.00203;$ $a_5=-0.0039; a_6=-0.0028; a_7=-0.00159; a_8=-0.0004;$ $a_9=-0.002; a_{10}=-0.0018; a_{11}=-0.0015; a_{12}=-0.0012;$ $a_{13}=-0.0009; a_{14}=-0.00065; a_{15}=-0.00036;$ $a_{16}=-0.00008; x=-10:10\ and\ t=0:0.01s.$



(a) (b)

Figure 16. (a) the 3D and (b) the 2D for Eq. (4.16) with M=32 at $\alpha=2, c=1, \beta=2, \omega=1,$ $a_1=0.0066, a_2=-0.0087, a_3=-0.00671, a_4=-0.00203,$ $a_5=-0.0039, a_6=-0.0028, a_7=-0.00159, a_8=-0.0004,$ $a_9=-0.002, a_{10}=-0.0018, a_{11}=-0.0015, a_{12}=-0.0012,$ $a_{13}=-0.0009, a_{14}=-0.00065, a_{15}=-0.00036, a_{16}=-0.00008,$ $a_{17}=-0.0011, a_{18}=-0.001, a_{19}=-0.0009, a_{20}=-0.0009,$ $a_{21}=-0.0008, a_{22}=-0.0007, a_{23}=-0.0007,$ $a_{24}=-0.00058, a_{25}=-0.00051, a_{26}=-0.0004,$ $a_{27}=-0.00036, a_{28}=-0.00029, a_{29}=-0.0002,$ $a_{30}=-0.00015, a_{31}=-0.00007, a_{32}=-0.000005,$ $x=-10:10\ and\ t=0:0.01s.$

The following table illustrates the comparison between the numerical solutions by HWM and analytic soliton solutions by RBSODEM

Table 3. Comparison between solutions of RBSODEM and HWM at t=0.01.

x	At $t = 0.01s$.				
	RBSODEM	M=4	M=8	M=16	M=32
-0.6	0.826434	0.8196	0.8196	0.8196	0.8196
-0.4	0.751701	0.7476	0.7476	0.7476	0.7476
-0.2	0.676933	0.6756	0.6756	0.6756	0.6756
0	0.603604	0.6036	0.6036	0.6036	0.6036
0.2	0.533073	0.531443	0.531386	0.531373	0.53137
0.4	0.466495	0.459069	0.458971	0.458947	0.458941
0.6	0.404751	0.386895	0.38676	0.386725	0.386716
0.8	0.348424	0.315209	0.315037	0.314993	0.314982
1	0.297805	0.244172	0.243968	0.243917	0.243905

Table 4. Comparison between solutions of RBSODEM and HWM at t=0.1.

x	At $t = 0.1s$.				
	RBSODEM	M=4	M=8	M=16	M=32
-0.6	0.859659	0.852	0.852	0.852	0.852
-0.4	0.785424	0.78	0.78	0.78	0.78
-0.2	0.710487	0.708	0.708	0.708	0.708
0	0.636333	0.636	0.636	0.636	0.636
0.2	0.564386	0.563956	0.563936	0.563928	0.563927
0.4	0.495902	0.491625	0.491554	0.491535	0.49153
0.6	0.431893	0.419331	0.419209	0.419178	0.419171
0.8	0.373076	0.3474	0.347239	0.347199	0.347189
1	0.31987	0.276052	0.275862	0.275815	0.275803

5.Conclusion

This research focuses on deriving new different forms of soliton solutions for the tunnel diode model governed by the LWE equation. Using innovative solitary wave techniques, such as the ESEM and RBSODEM techniques. Additionally, the numerical HWM method, recognized for its simplicity, speed, and efficiency, has been employed to obtain accurate numerical solutions. By applying the proposed techniques, various forms of soliton solutions have been identified such as periodic parabolic soliton solution, bright soliton solution, dark soliton solution, kink soliton solution. To validate these solutions, the numerical HWM method was used to generate identical numerical results. Our findings introduce new solutions compared to previous studies [16-20], which utilized alternative methods for solving this model. Furthermore, 2D and 3D graphical representations created using Mathematica and Maple programs illustrate the agreement between analytical and numerical solutions. These visualizations also reveal new dynamic properties of the signals associated with the tunnel diode model, offering deeper insights into its behaviour.

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